

(REVISED COURSE)

QP Code :11932

(3 Hours)

Total Marks : 80

N. B. : (1) Q. No.1 is compulsory.

(2) Attempt any three questions from question no.2 to question no.6.

(3) Figures to the right indicate full marks.

1. (a) If $\tanh x = \frac{2}{3}$, find the value of x and then $\cosh 2x$ 3

(b) If $u = \tan^{-1}\left(\frac{y}{x}\right)$, Find these value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ 3

(c) if $x = r \cos \theta$, $y = r \sin \theta$ 3

Find $\frac{\partial(x,y)}{\partial(r,\theta)}$

(d) Prove that $\operatorname{logsec} x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \dots$ 3

(e) Show that every square matrix can be uniquely expressed as the sum of Hermitian matrix and a skew Hermitian matrix. 4

(f) Find the n^{th} derivative of $y = \sin x \sin 2x \sin 3x$ 4

2. (a) Solve the equation $x^6+1=0$ 6

(b) Reduce the matrix to normal form and find its rank, where 6

$$A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$

(c) State and prove Eulers theorem for a homogeneous function in two variables: Hence verify the Eulers theorem for 8

$$u = \frac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$$

3. (a) Test the consistency of the following equations and solve them if they are consistent. 6

$$2x - y + z = 8, \quad 3x - y + z = 6$$

$$4x - y + 2z = 7, \quad -x + y - z = 4$$

(b) Find the stationary values $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ 6

(c) Separate into real and imaginary parts of $\sin^{-1}(e^{i\theta})$ 8

4. (a) If $x = uv, y = \frac{u}{v}$ prove that $J.J = 1$ 6

(b) Show that for real values of a and b, $e^{2 ai \cot^{-1} b} \left[\frac{bi-1}{bi+1} \right]^{-a} = 1$ 6

(c) Solve the following equations by Gauss-seidel method 8
 $27x + 6y - z = 85$
 $6x + 15y + 2z = 72$
 $x + y + 54z = 110$

5. (a) Expand $\cos^7 \theta$ in a series of cosines of multiple of θ 6

(b) If $\lim_{x \rightarrow 0} \frac{a \sinh x + b \sin x}{x^3} = \frac{5}{3}$, find a and b 6

(c) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$ then prove that $(1-x^2) y_{n+1} - (2n+1) x y_n - n^2 y_{n-1} = 0$ 8

6. (a) Examine whether the vectors 6
 $x_1 = [3, 1, 1]$ $x_2 = [2, 0, -1]$
 $x_3 = [4, 2, 1]$ are linearly independent.

(b) If $u = f(x-y, y-z, z-x)$ then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ 6

(c) Fit a straight line for the following data 8

x	1	2	3	4	5	6
y	49	54	60	73	80	86